Published by Institute of Physics Publishing for SISSA/ISAS
Received: *December 23, 2001*
Accepted: *February 27, 2002*

A new wrinkle on the enhançon

Dumitru Astefanesei[a] and Robert C. Myers[ab*]

[a] *Department of Physics, McGill University*
Montréal, Québec H3A 2T8, Canada
[b] *Perimeter Institute for Theoretical Physics*
20 Erb Street West, Waterloo, Ontario N2L 1T2, Canada and
Department of Physics, University of Waterloo
Waterloo, Ontario N2L 3G1, Canada
E-mail: astefand@hep.physics.mcgill.ca, rcm@hep.physics.mcgill.ca

Abstract: We generalize the basic enhançon solution of Johnson, Peet and Polchinski [1] by constructing solutions without spherical symmetry. A careful consideration of boundary conditions at the enhançon surface indicates that the interior of the supergravity solution is still flat space in the general case. We provide some explicit analytic solutions where the enhançon locus is a prolate or oblate sphere.

Keywords: Superstrings and Heterotic Strings, D-branes, Brane Dynamics in Gauge Theories.

[*] On leave from McGill University.

Contents

1. Introduction

One of the important lessons of the second string revolution was that string theory is more than just a theory of strings. That is, branes which are extended in varying numbers of dimensions play an important role in certain situations. Brane expansion is a fascinating physical effect that has been uncovered more recently in this context [2]. The latter is a remarkable nonlocal effect which string theory seems to employ in a wide variety of settings to resolve singularities and/or regulate divergences. One framework where brane expansion plays an important role is for repulsons [3, 4, 5].

Repulsons are a particular class of supersymmetric solutions of the supergravity equations of motion which contain naked (timelike) singularities [3, 4, 5]. With brane probe calculations, one can clearly argue that the regions of the solutions near the repulson singularity are unphysical [1]. Actually the naive supergravity equations are no longer valid in this region as they do not account for the full set of low energy degrees of freedom. Due to the stringy effects, one finds that when the internal compactification manifold reaches the string scale, there is an enhanced gauge symmetry accompanied by a massless vector supermultiplet of fields. As a result, the constituent branes become delocalized over a surface and the interior region of the solution is modified. For a large number of branes, the enhançon locus has macroscopic size and the modified solutions can still be studied with the framework of supergravity [1, 6, 7].

In the simplest case of the four-dimensional enhançon, one can think of the underlying system as a nonabelian SU(2) gauge theory coupled to gravity, however, in the present context, some of the relevant parameters are string scale and producing a reliable low energy theory for detailed calculations is difficult. Some progress has recently been made in this direction [8] by applying the Type IIa/heterotic duality [9]. In any event, the enhançon solution would be a BPS monopole in this underlying theory. The supergravity solution of [1] describes roughly the case where the energy of the underlying gauge fields is focused in a spherical shell at the enhançon radius. While there are no precisely spherical solutions for higher monopole charges [10], one can be confident that there are roughly spherical ones [11]. However, such pseudo-spherical symmetry would be at an exceptional point in the full moduli space of charge N monopoles. Hence the analogy would indicate that we should be able to deform the enhançon solution away from spherical symmetry. This is the topic of the present investigation. Rather than working with a microscopic theory, we will be analyzing the enhançon solution and its deformations using the low energy supergravity. In particular, we will be using tools developed in classical general relativity to study junctions [12, 13]. As in [7, 14, 15], we find that these low energy techniques still seem to be able to faithfully reproduce much of the stringy/braney physics of the enhançon.

The paper is organized as follows: in section 2, we review the relevant repulson solutions which naively describe the background solution generated by a collection of D(p+4)-branes wrapped on a K3 surface. Our discussion is general in that it allows for an arbitrary distribution of source branes. We then probe these generalized solutions with a test D(p+4)-brane, which allows us to identify the enhançon locus for these background configurations. In section 3, we examine the junction conditions for a cut-and-paste procedure where a new interior region is used to replace the repulson geometry inside the enhançon locus. We find gluing on a flat-space interior is consistent with the preceding probe calculations. In section 4, we examine the junction conditions in the enhançon solution in more detail and show that the source reproduces precisely the behavior of a distribution of wrapped D($p+4$)-branes. In section 5, we apply our results to two explicit cases of nonspherical enhançon solutions. Finally, we end with a brief discussion of our results in section 6.

2. Brane probes

As a warm-up exercise, we begin by examining probe branes moving in the supergravity background generated by a collection of wrapped D($p+4$)-branes. In the string frame, the ten-dimensional background metric for this system is:

$$ds^2 = Z_p^{-1/2} Z_{p+4}^{-1/2}\, \eta_{\mu\nu} dx^\mu dx^\nu + Z_p^{1/2} Z_{p+4}^{1/2}\, dx^i dx^i + Z_p^{1/2} Z_{p+4}^{-1/2}\, ds^2_{\mathrm{K3}}\,, \tag{2.1}$$

where ds^2_{K3} is the metric on the K3 surface with a fixed volume V_{K3}. Our conventions with respect to indices will be: $\{A, B \in 0, 1, \ldots, 9\}$ cover the entire spacetime, $\{\mu, \nu \in 0, 1, \ldots p\}$ indicate directions along the unwrapped world-volume, $\{i, j \in p+1, \ldots, 5\}$ cover the directions transverse to the branes, and $\{a, b \in 6, 7, 8, 9\}$ indicate K3 directions. We also adopt standard conventions such that Newton's constant is given by $16\pi G = (2\pi)^7 g_s^2\, \ell_s^8$ — see, e.g., [16]. The dilaton and Ramond-Ramond (RR) potentials for the solution are:

$$\begin{aligned}
e^{2\Phi} &= Z_p^{(3-p)/2} Z_{p+4}^{-(p+1)/2} \\
C^{(p+5)} &= Z_{p+4}^{-1}\, dx^0 \wedge \cdots \wedge dx^p \wedge \varepsilon_{\mathrm{K3}} \\
C^{(p+1)} &= Z_p^{-1}\, dx^0 \wedge \cdots \wedge dx^p \,,
\end{aligned} \tag{2.2}$$

where $\varepsilon_{\mathrm{K3}}$ denotes the volume four-form on K3, normalized such that $\int \varepsilon_{\mathrm{K3}} = \mathrm{V}_{\mathrm{K3}}$. Note that these RR potentials do *not* vanish asymptotically, however, this will be a convenient gauge choice.

The two harmonic functions may be written as

$$Z_{p+4} = 1 + f(x^i) \qquad Z_p = 1 - \frac{\mathrm{V}_*}{\mathrm{V}_{\mathrm{K3}}} f(x^i)\,, \tag{2.3}$$

where $\mathrm{V}_* = (2\pi\ell_s)^4$ and f is a (positive) harmonic function which vanishes in the asymptotic region, i.e., $\partial^i\partial_i f = 0$ (up to localized source terms) and $f \to 0$ as $(x^i)^2 \to \infty$. As a final remark, we note that the running K3 volume is given by

$$\mathrm{V}(x^i) = \frac{Z_p(x^i)}{Z_{p+4}(x^i)}\, \mathrm{V}_{\mathrm{K3}}\,. \tag{2.4}$$

The spherically symmetric solution for N wrapped D$(p+4)$-branes is given by [1]

$$f(r) = c_{(p+4)} \frac{N g_s \ell_s^{3-p}}{r^{3-p}}\,, \tag{2.5}$$

where $r^2 = (x^i)^2$. We have also introduced the standard normalization constant [17]: $c_{(p+4)} = \Gamma((3-p)/2)/(4\pi)^{(p-1)/2}$. By considering a general harmonic function f, we are allowing for more general distributions of D$(p+4)$-branes. As it stands this repulson [3, 4, 5] solution contains a naked singularity at $f(x^i) = \mathrm{V}_{\mathrm{K3}}/\mathrm{V}_*$. However, given the results of ref. [1], one expects that the region near this singularity is unphysical. As in this previous work, we will determine the boundary of the region of validity by probing the spacetime with a test D$(p+4)$-brane.

The effective world-volume action of a single wrapped D$(p+4)$-brane in the above background is

$$\begin{aligned}
S = &- \int_\Sigma d^{p+1}\sigma\, e^{-\Phi(x^i)} (\tau_{(p+4)} \mathrm{V}(x^i) - \tau_p)(-\det P[G]_{\mu\nu})^{1/2} + \\
&+ \tau_{(p+4)} \int_{\Sigma\times \mathrm{K3}} C^{(p+5)} - \tau_p \int_\Sigma C^{(p+1)}\,,
\end{aligned} \tag{2.6}$$

where Σ is the unwrapped part of the brane's world-volume with coordinates σ^μ with $\mu = 0, 1, \ldots, p$. $P[G]_{\mu\nu}$ denotes the pull-back of the string-frame metric to this part of the world-volume. We have adopted the conventions convenient for working with supergravity solutions, as described in ref. [2]. In particular, note that the coefficients of the Wess-Zumino terms in eqn. (2.6) are τ_p including a factor of $1/g_s$, i.e., the Dp-brane tension and charge are related by $\tau_p = \mu_p$. As usual, we have

$$\frac{\tau_p}{\tau_{p+4}} = (2\pi\ell_s)^4 = \mathrm{V}_*\,. \tag{2.7}$$

The above result (2.6) includes the negative contributions to both the tension and the $(p+1)$-form RR charge terms which arise from wrapping the D$(p+4)$-brane on K3 as described in [1] and [18]–[21].

Implicitly, we have chosen static gauge, leaving the probe to move in the directions transverse to the K3 while freezing and smearing the degrees of freedom on K3. Hence the world-volume coordinates σ^μ are aligned with the first $(p+1)$ spacetime coordinates, and there are shape fluctuations in the transverse directions:

$$\sigma^\mu = x^\mu \ , \qquad x^i = x^i(\sigma^\mu) \, . \tag{2.8}$$

Hence the induced metric on the effective $(p+1)$-dimensional world-volume is given by

$$P[G]_{\mu\nu} = G_{AB} \frac{\partial x^A}{\partial \sigma^\mu} \frac{\partial x^B}{\partial \sigma^\nu} = G_{\mu\nu} + G_{ij} \frac{\partial x^i}{\partial \sigma^\mu} \frac{\partial x^j}{\partial \sigma^\nu} \, . \tag{2.9}$$

Expanding the action (2.6) to quadratic order in derivatives then yields

$$S = \int d^{p+1}\sigma \, \left(T(x^i, \partial_\mu x^i) - U(x^i) \right) . \tag{2.10}$$

After a brief calculation, we find that the potential U vanishes[1] while the kinetic term becomes:

$$T(x^i) = -\frac{\tau_p}{2} \left(\frac{\mathrm{V}(x^i)}{\mathrm{V}_*} - 1 \right) \partial_\mu x^i \partial^\mu x^i \, . \tag{2.11}$$

The vanishing potential is, of course, a reflection of the fact that the probe brane respects the supersymmetry of the background configuration. From the kinetic term, we read off the effective tension of the probe p-brane with

$$\tau(x^i) \propto \frac{\mathrm{V}(x^i)}{\mathrm{V}_*} - 1 \, . \tag{2.12}$$

This confirms the expected result that, irrespective of the distribution of sources generating the background geometry, we find an enhançon locus precisely where the K3 volume reaches V_*. Combining eqs. (2.3) and (2.4), this surface is defined by the equation

$$f(x^i) = f_* \equiv \frac{1}{2} \left(\frac{\mathrm{V}_{\mathrm{K3}}}{\mathrm{V}_*} - 1 \right) . \tag{2.13}$$

Inside this surface ($f(x^i) > f_*$), the effective probe tension (2.12) would be negative, and following ref. [1], we interpret this as an indication that this region of the supergravity solution is spurious. That is, eq. (2.1) does not describe the correct background spacetime that would generated in string theory when one assembles the corresponding collection of wrapped D$(p+4)$-branes.

[1]Note that we have precisely $U = 0$ rather than some other constant because of the convenient choice of gauge for the RR fields in eq. (2.2).

3. Junction conditions

The probe calculation suggests that the interior of the repulson solution (i.e., for $f(x^i) > f_*$) should be excised and replaced by a new solution. Following refs. [7, 14, 15], we can use the techniques of classical general relativity to investigate the matching conditions in detail. When we join the exterior and interior solutions at the enhançon locus $f(x^i) = f_*$, we match the geometry (as well as the dilaton and RR potentials) of the two solutions but in general there will be a discontinuity in the extrinsic curvature at this surface. The latter can be interpreted as a δ-function source of stress-energy [12, 13] produced by the delocalized branes spread out across the excision surface.

To properly identify this discontinuity as a stress-energy, the calculations are performed in the Einstein frame. Hence we perform the standard conformal rescaling of the string-frame metric given above: $ds^2_{\rm E} = e^{-\Phi/2} ds^2_{\rm S}$. The Einstein-frame metric for the wrapped D$(p+4)$-branes is then

$$ds^2 = Z_p^{\beta_1} Z_{p+4}^{\alpha_1} \eta_{\mu\nu} dx^\mu dx^\nu + Z_p^{\beta_2} Z_{p+4}^{\alpha_2} \hat{g}_{ij} dx^i dx^j + Z_p^{\beta_2} Z_{p+4}^{\alpha_1} ds^2_{\rm K3}\,, \tag{3.1}$$

where the exponents are: $\alpha_1 = (p-3)/8$, $\beta_1 = (p-7)/8$, $\alpha_2 = \alpha_1 + 1$ and $\beta_2 = \beta_1 + 1$. In the following, we will denote the components of the Einstein-frame metric as simply g_{AB}. Above, we have introduced a general *flat-space* metric $\hat{g}_{ij}$ in the transverse space, to explicitly exhibit the coordinate invariance of the following results.

In general, the new interior solution would be a solution of the same form as given in eqs. (2.2) and (3.1) but with modified harmonic functions. Hence we write the Einstein metric in the interior region as

$$ds^2 = H_p^{\beta_1} H_{p+4}^{\alpha_1} \eta_{\mu\nu} dx^\mu dx^\nu + H_p^{\beta_2} H_{p+4}^{\alpha_2} \hat{g}_{ij} dx^i dx^j + H_p^{\beta_2} H_{p+4}^{\alpha_1} ds^2_{\rm K3}\,, \tag{3.2}$$

In order to match the interior and exterior geometries, we must impose the boundary conditions that $H_p = Z_p$ and $H_{p+4} = Z_{p+4}$ at the boundary surface $f = f_*$ — note then that the harmonic functions are all constant on this surface. Of course, the same harmonic functions appear in the solution for the dilaton and RR potentials, in analogy with eq. (2.2). Now, as the branes are delocalized at the enhançon locus, the interior harmonic functions must satisfy Laplace's equation, $\partial^i \partial_i H = 0$, with *no source terms.* Given the above boundary condition, and assuming that the enhançon locus is a closed surface in the transverse directions x^i, it is a straightforward exercise to show that the only solutions of Laplace's equation are constants. Hence we have that

$$H_p = Z_p|_{f=f_*} = \frac{1}{2}\left(1 + \frac{\rm V_{K3}}{\rm V_*}\right), \qquad H_{p+4} = Z_{p+4}|_{f=f_*} = \frac{1}{2}\left(1 + \frac{\rm V_*}{\rm V_{K3}}\right), \tag{3.3}$$

throughout the interior region. That is, just as in the spherically symmetric case, the interior geometry must be simply flat space for consistency with the supergravity equations of motion.

Now let us proceed with the calculation of the boundary stress-energy. From eq. (2.13), the enhançon locus is defined as the surface $f(x^i) = f_*$ in the exterior geometry, however, we will generalize our calculations slightly by performing the excision at $f(x^i) = f_{\rm ex}$ for an arbitrary (positive) constant $f_{\rm ex}$. In this case, the harmonic functions in the interior region become $H_p = Z_p|_{f_{\rm ex}}$ and $H_{p+4} = Z_{p+4}|_{f_{\rm ex}}$. For both the interior and exterior regions, we need to construct an outward-pointing unit normal vector to this surface, i.e., $n_{\pm A}$ such that

$$g^{AB} n_{\pm A} n_{\pm B}|_{f=f_{\rm ex}} = 1\,. \tag{3.4}$$

By definition then, the extrinsic curvature of the boundary for each region is given by

$$K^{\pm}_{AB} = h_A{}^C h_B{}^D\, \nabla_C n_{\pm D}\,, \tag{3.5}$$

where $h_{AB} = g_{AB} - n_A n_B$ is the intrinsic metric[2] of $f(x^i) = f_{\rm ex}$. The gradient of the function f is orthogonal to the matching surface, but in general we have $g^{AB}\partial_A f \partial_B f = g^{ij}\partial_i f \partial_j f = F(x^i) \neq 1$. Hence we can define

$$n_{\pm\mu} = 0,, \qquad n_{\pm i} = \pm\frac{1}{\sqrt{F}}\,\partial_i f\,, \qquad n_{\pm a} = 0\,. \tag{3.6}$$

In choosing the sign above, we have used the assumption, introduced already at eq. (2.3), that f is positive and vanishes asymptotically.

The boundary stress-energy is related to the discontinuity in the extrinsic curvature across the junction [12, 13]:

$$\begin{aligned} 8\pi G\, S_{AB} &= \gamma_{AB} - h_{AB}\,\gamma^C{}_C \\ &= (K^+ + K^-)_{AB} - h_{AB}\,(K^+ + K^-)^C{}_C\,. \end{aligned} \tag{3.7}$$

As an example, we explicitly calculate $K^+_{\mu\nu}$:

$$\begin{aligned} K^+_{\mu\nu} &= h_\mu{}^\rho h_\nu{}^\sigma \nabla_\rho\, n_{+\sigma} = \delta_\mu{}^\rho \delta_\nu{}^\sigma (\partial_\rho\, n_{+\sigma} - n_{+A}\,\Gamma^A{}_{\rho\sigma}) \\ &= -n_{+i}\,\Gamma^i_{\mu\nu} = \frac{1}{2} n_{+i} g^{ij} (\partial_j\, g_{\mu\nu}) \\ &= \frac{1}{2} n_{+i} g^{ij} \eta_{\mu\nu}\left[\alpha_1\, Z_{p+4}^{\alpha_1 - 1} Z_p^{\beta_1}\, \partial_j f - \frac{\mathrm{V}_*}{\mathrm{V}_{\rm K3}}\beta_1\, Z_{p+4}^{\alpha_1} Z_p^{\beta_1 - 1}\, \partial_j f\right] \\ &= \frac{1}{2}\left[\frac{\alpha_1}{Z_{p+4}} - \frac{\mathrm{V}_*}{\mathrm{V}_{\rm K3}}\frac{\beta_1}{Z_p}\right]\sqrt{F}\, h_{\mu\nu}\,. \end{aligned} \tag{3.8}$$

With a similar calculation, one finds

$$K^+_{ab} = \frac{1}{2}\left[\frac{\alpha_1}{Z_{p+4}} - \frac{\mathrm{V}_*}{\mathrm{V}_{\rm K3}}\frac{\beta_2}{Z_p}\right]\sqrt{F}\, h_{ab}\,. \tag{3.9}$$

The calculation of K^+_{ij} is slightly more complicated because g_{ij} depends on the transverse

[2]Of course, this metric is the same irrespective of whether one chooses n_+ or n_-.

coordinates both through the harmonic functions and $\hat{g}_{ij}$. One finds:

$$\begin{aligned}
K^{+}_{ij} &= h_i{}^k h_j{}^l \nabla_k\, n_{+l} = h_i{}^k h_j{}^l (\partial_k\, n_{+l} - n_{+A}\, \Gamma^A{}_{kl}) \\
&= h_i{}^k h_j{}^l \left[\partial_k\, n_{+l} - \frac{1}{2} n_{+n} g^{nm} (\partial_l\, g_{mk} + \partial_k\, g_{ml} - \partial_m\, g_{lk})\right] \\
&= h_i{}^k h_j{}^l \Bigg[\partial_k\, n_{+l} - \frac{1}{2} n_{+n} \hat{g}^{nm} (\partial_l \hat{g}_{mk} + \partial_k \hat{g}_{ml} - \partial_m \hat{g}_{lk}) + \\
&\qquad + \frac{1}{2} n_{+n} g^{nm} g_{lk} \left[\frac{\alpha_2}{Z_{p+4}} - \frac{\mathrm{V}_*}{\mathrm{V}_{\mathrm{K3}}} \frac{\beta_2}{Z_p}\right] \partial_m f \Bigg] \\
&= h_i{}^k h_j{}^l \hat{\nabla}_k\, n_{+l} + \frac{1}{2} \left[\frac{\alpha_2}{Z_{p+4}} - \frac{\mathrm{V}_*}{\mathrm{V}_{\mathrm{K3}}} \frac{\beta_2}{Z_p}\right] \sqrt{F}\, h_{ij}\,. \qquad (3.10)
\end{aligned}$$

Above, the third line was simplified using $h_i{}^l \partial_l f \propto h_i{}^l n_l = 0$, and $\hat{\nabla}_k$ denotes the covariant derivative with respect to the transverse metric $\hat{g}_{ij}$. As the harmonic functions are constant in the interior region, the calculations of the extrinsic curvature are somewhat simpler. After a short calculation, one finds:

$$\begin{aligned}
K^{-}_{\mu\nu} &= 0 = K^{-}_{ab}\,, \\
K^{-}_{ij} &= h_i{}^k h_j{}^l \hat{\nabla}_k\, n_{-l} = -h_i{}^k h_j{}^l \hat{\nabla}_k\, n_{+l}\,. \qquad (3.11)
\end{aligned}$$

Given these results, a straightforward calculation yields the surface stress tensor as:

$$8\pi G\, S_{\mu\nu} = -\frac{\sqrt{F}}{2} \left(\frac{1}{Z_{p+4}} - \frac{\mathrm{V}_*}{\mathrm{V}_{\mathrm{K3}}} \frac{1}{Z_p}\right) h_{\mu\nu}\,, \qquad (3.12)$$

$$8\pi G\, S_{ab} = -\frac{\sqrt{F}}{2} \frac{1}{Z_{p+4}}\, h_{ab}\,, \qquad (3.13)$$

$$8\pi G\, S_{ij} = 0\,. \qquad (3.14)$$

This result is precisely in accord with the probe brane calculations, as was previously observed in [7, 14]. First, the stresses in the transverse directions always vanish. This was required for consistency as the wrapped D$(p+4)$-branes form a BPS configuration for any distribution of (parallel) branes. Hence there are no stresses required to support a shell of any shape or size. Second, the K3 components of the surface stress-energy are determined by a single, *positive* effective tension

$$T_{\mathrm{K3}} = \frac{\sqrt{F}}{16\pi G} \frac{1}{Z_{p+4}}\,. \qquad (3.15)$$

Note that this tension only depends on the harmonic function for the D$(p+4)$-branes, as is appropriate because there are only pure D$(p+4)$-branes wrapped there. The surface stress-energy tensor has a similar form in the effective p-brane directions (i.e., x^μ) with an effective tension

$$T_p = \frac{\sqrt{F}}{16\pi G} \left(\frac{1}{Z_{p+4}} - \frac{\mathrm{V}_*}{\mathrm{V}_{\mathrm{K3}}} \frac{1}{Z_p}\right). \qquad (3.16)$$

Note that this effective tension vanishes precisely at the enhançon locus $f = f_*$, where $Z_p/Z_{p+4} = \mathrm{V}_*/\mathrm{V}_{\mathrm{K3}}$. For $f = f_{\mathrm{ex}} < f_*$, the tension is positive in accord with the probe brane results, while for $f = f_{\mathrm{ex}} > f_*$, the result is negative. Hence our supergravity calculations are consistent with the stringy phenomenon of an enhanced gauge symmetry appearing precisely at the duality volume V_*.

Note that in both cases, the effective tensions are only local quantities. First, they depend on the 'size' of the shell or the choice of f_{ex}, which modifies the value of the harmonic functions at the boundary. Secondly, they vary with the position on the shell since, although the harmonic functions are constant across the boundary surface, in general $F = g^{ij}\partial_i f \partial_j f$ will vary. This does not mean that the tension of the constituent branes varies from point to point on the shell but rather that the density of branes is not constant over a shell with a general shape. The spherically symmetric case (2.5) is of course an exception to this statement, where the tension and density of branes are constant over a shell of a given size. We examine this aspect of the delocalization of the branes in more detail in the next section.

4. Source calculations

In the second section, we have seen that the probe tension becomes negative inside the enhançon locus, therefore the probe cannot penetrate the hypersurface $f(x^i) = f_*$. This result was confirmed above by the calculation of the boundary stress tensor for the background supergravity solution. There we found that the effective tension (3.16) of a shell of some number N of wrapped D$(p+4)$-branes vanishes as the shell approaches the enhançon locus, and that this tension is negative if we try to construct a solution inside this surface, i.e., with $f_{\mathrm{ex}} > f_*$. As in refs. [1, 6], we interpret the minimal solution, where the excision is made at $f(x^i) = f_*$, as a shell of branes with zero tension smeared across this surface.

We can make this relation more precise by showing that the stress-energy of the shell precisely matches that of N wrapped D$(p+4)$-branes distributed across the enhançon locus, following refs. [7, 14, 15]. As mentioned above, we will find that the branes are not uniformly distributed. However, the density of branes is consistently reproduced in the stress-energy calculations, and in considering how the shell acts as a source for the dilaton and RR fields.

We begin with calculating the brane density by comparing the stress-energy of a collection of source branes with the shell stress-energy given in eqs. (3.12) and (3.13). First, we must consider how to eliminate the 'radial' δ-function in the stress-energy of a shell of branes. Towards this end, note that in the vicinity of the excision surface, the Einstein-frame metric can be rewritten

$$ds^2 = h_{AB}dy^A dy^B + g_{ff}\, df^2\,, \tag{4.1}$$

where we use the harmonic function $f(x^i)$ as the coordinate normal to the $f(x^i) = f_{\mathrm{ex}}$ hypersurface. In this case, $g_{ff} = 1/F(x^i)$. Note that we have also introduced a set of intrinsic coordinates y^A which parameterize the positions on this surface, but their details will play no essential role below. Hence the boundary stress tensor of the shell should be compared with

$$S_{AB} = \int \sqrt{g_{ff}}df \sum_{\text{shell}} \left[-\frac{2}{\sqrt{-g}} \frac{\delta S_{\text{brane}}}{\delta g^{AB}} \right], \tag{4.2}$$

where the summation means that we should sum over the contributions of all of the constituent branes in the shell. The term in the square brackets is just the standard definition of the stress-energy, where the variation is made with respect to the Einstein-frame metric. As can be seen in eq. (2.6), only the Dirac-Born-Infeld part of the brane action explicitly contributes to the metric source.[3] In the Einstein frame, the DBI action for an individual wrapped D$(p+4)$-brane becomes

$$S_{DBI} = -\int_{\Sigma} d^{p+1}\sigma \; e^{\frac{p-3}{4}\Phi(x^i)} (\tau_{(p+4)} e^{\Phi(x^i)} \mathrm{V}_E(x^i) - \tau_p)(-\det P[g]_{\mu\nu})^{1/2}, \tag{4.3}$$

where $V_E(x^i) = \int_{\mathrm{K3}} d^4x \sqrt{\det P[g]_{ab}}$ is the K3 volume in Einstein frame. The Einstein-frame and string-frame volumes of the K3 surface are related by $\mathrm{V}_E(x^i) = \mathrm{V}(x^i)e^{-\Phi(x^i)}$. If, for simplicity, we begin by considering the components of the stress-energy in the K3 directions, a short calculation combining eqs. (4.2) and (4.3) yields

$$S_{ab} = -\tau_{(p+4)}\, \rho(y^i)\, e^{\frac{p+1}{4}\Phi(y^i)}\, g_{ab}\,. \tag{4.4}$$

Here we have assumed that the sources are smeared out over the surface $f(x^i) = f_{\text{ex}}$, and so we have replaced the sum over constituent branes in eq. (4.2) by a smooth density $\rho(y^i)$. Using $16\pi G = (2\pi)^7 g_s^2\, \ell_s^8$ and $\tau_{(p+4)} = \frac{2\pi}{(2\pi\ell_s)^{p+5} g_s}$, one finds upon comparing eqs. (3.13) and (4.4) that the density is given by

$$\rho(y^i) = \frac{(2\pi l_s)^{p-3}}{g_s} \sqrt{\hat{g}^{ij}\partial_i f \partial_j f}\; (Z_{p+4}^{\alpha_2} Z_p^{\beta_2})^{\frac{p-4}{2}}, \tag{4.5}$$

where again this expression is evaluated on the excision surface $f(x^i) = f_{\text{ex}}$. Note then that Z_{p+4} and Z_p are constants in this expression, i.e., they are independent of the intrinsic coordinates y^i. Hence any variation in the density comes from the factor involving the gradient of f.

One simple check of the above result (4.5) is to confirm that $\rho(y^i)$ takes the usual form for the spherical enhançon [1]. Of course, in this situation, spherical symmetry requires the brane density to be a constant. Recall that the standard harmonic function for this case was given in eq. (2.5), and the intrinsic coordinates y^i could be taken to be the angular coordinates in a spherical polar coordinate system. Applying eq. (4.5) yields

$$\rho_{sph} = \frac{N}{\Omega_{4-p} r_{ex}^{4-p}}\; (Z_{p+4}^{\alpha_2} Z_p^{\beta_2})^{\frac{p-4}{2}} \Big|_{f=f_{\text{ex}}}, \tag{4.6}$$

where $\Omega_{4-p} = 2\pi^{(5-p)/2}/\Gamma((5-p)/2)$ is the area of a unit $(4-p)$-sphere, and r_{ex} is the excision radius where $f(r) = f_{\text{ex}}$. Hence the first factor corresponds to the density of N

[3] As noted in ref. [7], the curvature couplings in the Wess-Zumino action, which do play an important role in the physics of the enhançon, do not contribute a source term to Einstein's equations.

branes smeared over a $(4-p)$-sphere of radius r_{ex}, while the second factor corrects the sphere volume to be the proper volume of the sphere in the Einstein-frame metric (3.1).

A further check on our result for the brane density comes from analyzing the components of the boundary stress-energy along the effective p-brane directions. In this case, the variation of the DBI action (4.3) yields

$$\begin{aligned} S_{\mu\nu} &= -\rho(y^i)\left(\tau_{(p+4)}e^{\frac{p+1}{4}\Phi(y^i)} - \frac{\tau_p}{V_E(y^i)}e^{\frac{p-3}{4}\Phi(y^i)}\right) h_{\mu\nu} \\ &= -\tau_{(p+4)}\rho(y^i)e^{\frac{p+1}{4}\Phi(y^i)}\left(1-\frac{\mathrm{V}_*}{V(y^i)}\right) h_{\mu\nu}\,, \end{aligned} \tag{4.7}$$

where we have used $\mathrm{V}(x^i) = \mathrm{V}_E(x^i)e^{\Phi(x^i)}$ and eq. (2.7) in simplifying the second line. For comparison purposes, note that we can write eq. (3.12) as

$$S_{\mu\nu} = -\frac{\sqrt{F}}{16\pi G}\frac{1}{Z_{p+4}}\left(1-\frac{\mathrm{V}_*}{V(y^i)}\right) h_{\mu\nu} \tag{4.8}$$

using eq. (2.4). Thus, agreement between these two expressions yields precisely the same brane density as was determined above.

As a further consistency check of eq. (4.5), we also consider the source that the shell provides for the dilaton. Here we generalize the methods presented in ref. [7] to situations without spherical symmetry.[4] The essential observation is to replace the usual radial coordinate by the normal coordinate f, as in eq. (4.1). Then let us write the harmonic functions for the complete solution where an excision is made at the surface $f = f_{\mathrm{ex}}$ as

$$\begin{aligned} H_{p+4}(f) &= Z_{p+4}(f_{\mathrm{ex}}) + \theta(f_{\mathrm{ex}}-f)\,\left(Z_{p+4}(f) - Z_{p+4}(f_{\mathrm{ex}})\right)\;, \\ H_p(f) &= Z_p(f_{\mathrm{ex}}) + \theta(f_{\mathrm{ex}}-f)\,\left(Z_p(f) - Z_p(f_{\mathrm{ex}})\right)\;. \end{aligned} \tag{4.9}$$

Recall that f is positive and vanishes asymptotically. Note that f continues to serve as a useful coordinate at least slightly inside the excision surface, which will suffice for our purposes. Now differentiating with respect to f, one finds

$$\partial_f^2 H_{p+4} = -\delta(f-f_{\mathrm{ex}})\,, \qquad \partial_f^2 H_p = \frac{\mathrm{V}_*}{\mathrm{V}_{\mathrm{K3}}}\delta(f-f_{\mathrm{ex}})\,, \tag{4.10}$$

where from eq. (2.3), we have $\partial_f Z_{p+4} = 1$ and $\partial_f Z_p = -\mathrm{V}_*/\mathrm{V}_{\mathrm{K3}}$. Hence δ-functions like these will appear in evaluating the equation of motion for the dilaton, but this simply reflects the fact that there is a singular source, i.e., the shell of branes, in the full equations for the bulk supergravity fields coupled to the worldvolume action of the D-branes. To match the source, we must simply isolate the most singular terms in the bulk field equations.

The full coupled dilaton equation of motion is

$$\nabla^2\Phi \simeq -16\pi G\sum_{\mathrm{shell}}\frac{1}{\sqrt{-g}}\frac{\delta}{\delta\Phi}S_{\mathrm{brane}}\,, \tag{4.11}$$

[4] We are grateful to Amanda Peet for notes on her extension of the calculations in ref. [7] to arbitrary values of p.

where we have dropped the term on the left-hand side arising from the dilaton coupling to the RR fields, since it will not contribute to the δ-function. In evaluating this equation of motion, it is important to remember that this result was derived by varying the dilaton while holding the Einstein-frame metric fixed. Now from eq. (2.2), the dilaton solution is given by

$$\Phi = \ln\left(H_p^{\frac{3-p}{4}} H_{p+4}^{-\frac{p+1}{4}}\right) . \tag{4.12}$$

Hence given eq. (4.10), the most singular term on the left-hand side of eq. (4.11) is

$$g^{ff}\partial_f^2\Phi \simeq F\left(\frac{p+1}{4}\frac{1}{Z_{p+4}} - \frac{p-3}{4}\frac{\mathrm{V}_*}{\mathrm{V}_{\mathrm{K3}}Z_p}\right)\delta(f-f_{\mathrm{ex}}) . \tag{4.13}$$

Just as for the metric variations, only the DBI action contributes to the right-hand side dilaton equation (4.11). Hence, the dilaton source term is

$$-\frac{16\pi G}{\sqrt{-g}}\frac{\delta}{\delta\Phi}\sum_{\mathrm{shell}} S_{DBI} = 16\pi G\rho(y^i)\frac{1}{\sqrt{g_{ff}}}\left(\frac{p+1}{4}\tau_{(p+4)}e^{\frac{p+1}{4}\Phi} - \frac{p-3}{4}\frac{\tau_p}{V_E(y^i)}e^{\frac{p-3}{4}\Phi}\right)\delta(f-f_{\mathrm{ex}}) . \tag{4.14}$$

Now, given our experience from the analysis of the source for the boundary stress-energy, it is not hard to show that eqs. (4.13) and (4.14) agree if the brane density in the latter equation is given by precisely the expression in eq. (4.5).

Finally, we add that a similar analysis shows that the density in eq. (4.5) also reproduces the correct discontinuity in the RR fields, i.e., $C^{(p+5)}$ and $C^{(p+1)}$, although we will not present any of the details here.

5. Nonspherical enhançons

In this section, we consider two explicit examples of enhançon solutions without spherical symmetry. In the first case, we consider modifying the standard spherical solution [1] by the addition of a term involving a higher spherical harmonic function. In the second example, we construct a simple solution by introducing a new, nonspherically symmetric coordinate system in the transverse space.

5.1 Perturbing the spherical enhançon

One can easily modify the harmonic function (3.2) found in the spherical case by adding a higher spherical harmonic term. In this case, the solution for N wrapped D$(p+4)$-branes would be given by

$$f(r,\theta_i,\phi) = c_{(p+4)}\frac{Ng_s\ell_s^{3-p}}{r^{3-p}} + \frac{a}{r^{3-p+L}}\psi_L(\theta_i,\phi) , \tag{5.1}$$

where we have only labelled the angular function with the highest quantum number $L = l_{3-p}$ — the details of these functions may be found in appendix. Now as usual, the enhançon locus is the surface where the tension drops to zero, which is still given by eq. (2.13). For the case of the spherical enhançon, this surface is easily computed to be [1]

$$r_e^{3-p} = \frac{2\mathrm{V}_*}{\mathrm{V}_{\mathrm{K3}} - \mathrm{V}_*}c_{(p+4)}Ng_s\ell_s^{3-p} . \tag{5.2}$$

Now if we treat the higher harmonic term in eq. (5.1) as a small perturbation, i.e., if $a/r_e^{3-p+L} \ll 1$, then the enhançon surface will be slightly deformed to sit at

$$r_e + \delta r(\theta_i, \phi) \quad \text{where} \quad \delta r = \frac{1}{3-p} \frac{2\mathrm{V}_*}{\mathrm{V}_{\mathrm{K3}} - \mathrm{V}_*} \frac{a}{r_e^{2-p+L}} \psi_L(\theta_i, \phi) \,. \tag{5.3}$$

In this case, to leading order in the perturbation, the brane density is

$$\rho(r, \theta_i, \phi) = \rho_{sph}(r_e) \left(1 + (L + 3 - p) \frac{\delta r(\theta_i, \phi)}{r_e} \right) , \tag{5.4}$$

where $\rho_{sph}(r_e)$ is defined in eq. (4.6). Hence the modified density now varies across the enhançon locus. A few observations are: When averaged over the surface $\langle \delta r \rangle = 0$ so, as expected, the net number of branes in the surface remains at N. Further, the density is greater (smaller) than the spherical density in the regions where δr is positive (negative). Roughly, this indicates that the density of branes becomes concentrated in regions where the curvature of the surface is greater.

5.2 Prolate/Oblate enhançon

By considering the prolate or oblate spheroidal coordinates on the transverse flat coordinates [22], one can easily construct new analytic solutions describing nonspherical enhançons. For example, with three transverse spatial dimensions (i.e., we set $p = 2$ in the previous discussion and hence consider wrapped D6-branes), one defines

$$\begin{aligned} x^1 &= \sqrt{R^2 + k} \sin\theta_1 \cos\phi \,, \\ x^2 &= \sqrt{R^2 + k} \sin\theta_1 \sin\phi \,, \\ x^3 &= R \cos\theta \,, \end{aligned} \tag{5.5}$$

where k is a (nonzero) constant, ϕ is the angle in the x^1-x^2 plane and θ_1 is, roughly speaking, an angle away from the x^3-axis. Surfaces of constant R describe ellipses of rotation,

$$\frac{(x^1)^2 + (x^2)^2}{R^2 + k} + \frac{(x^3)^2}{R^2} = 1 \,. \tag{5.6}$$

If k=0, then one has just the standard polar coordinates on R^3. If $k > 0$ ($k < 0$), then eq. (5.6) describes an oblate (a prolate) sphere and this parameterization of flat space is called oblate (prolate) spheriodal coordinates [22]. With these coordinates, the flat space metric becomes

$$ds^2 = \frac{R^2 + k\cos^2\theta_1}{R^2 + k} dR^2 + (R^2 + k\cos^2\theta_1) d\theta_1^2 + (R^2 + k)\sin^2\theta_1 d\phi^2 \,. \tag{5.7}$$

One of the remarkable properties of this coordinate system is that Laplace's equation remains separable. If one considers a harmonic function without any angular dependence, the Laplace equation, $\nabla^2 f = 0$, reduces to

$$\partial_R[(R^2 + k)\partial_R f(R)] = 0 \,, \tag{5.8}$$

which has the following solutions

$$f(R)=\begin{cases} a/\sqrt{k}\,\mathrm{arccot}\,R/\sqrt{k} & k>0\,,\\ a/\sqrt{-k}\,\mathrm{arccoth}\,R/\sqrt{-k} & k<0\,,\end{cases} \tag{5.9}$$

where the constants of integration are chosen so that f is positive and vanishes asymptotically. For a system of N wrapped D6-branes, we may normalize a by comparing these results to that for a spherical enhançon, as given in eq. (2.5). The latter behavior will be dominant at large radius as can be seem by expanding eq. (5.9), which yields: $f\simeq a/R$ (for either sign of k). Hence we can set

$$a=\frac{N}{2}g_s\ell_s\,. \tag{5.10}$$

The enhançon surface is still given by eq. (2.13) and so the enhançon 'radius' becomes

$$R_e=\begin{cases} \sqrt{k}\cot\sqrt{k}f_*/a & k>0\,,\\ \sqrt{-k}\coth\sqrt{-k}f_*/a & k<0\,.\end{cases} \tag{5.11}$$

Applying eq. (4.5), the density of branes across this surface is given by

$$\rho=\frac{N}{2\pi}\frac{1}{\sqrt{(R_e^2+k\cos^2\theta_1)(R_e^2+k)}}\,(H_{p+4}^{-\alpha_2}H_p^{-\beta_2})\Big|_{R=R_e}\,. \tag{5.12}$$

Note that for k positive (negative), the density is smallest (greatest) at the x^3-axis and greatest (smallest) at the equator in the x^1-x^2 plane.

A final observation is that we could transform from the prolate/oblate spheroidal coordinates (R,θ_1,ϕ) to standard spherical polar coordinates (r,θ,ϕ) on R^3. While the general expression is not very illuminating, it is interesting to make an asymptotic expansion which yields

$$f(r,\theta)=\frac{a}{r}-\frac{a\,k}{3r^3}P_2(\cos\theta)+\frac{a\,k^2}{5r^5}P_4(\cos\theta)+O(r^{-7})\,. \tag{5.13}$$

Here $P_2(x)=(3x^2-1)/2$ and $P_4(x)=(35x^4-30x^2+3)/8$ are the second and fourth Legendre polynomials, respectively. Hence we produce a(n infinite) series of higher harmonics of the form considered in the previous subsection. This form (5.13) is useful in that it allows one to confirm the overall normalization constant a chosen in eq. (5.10).

This construction can easily be extended to four dimensions, i.e., with $p=1$ and wrapped D5-branes. The analogous prolate/oblate spheroidal coordinates are[5]

$$\begin{aligned} x^1 &= \sqrt{R^2+k}\sin\theta_1\cos\phi_1\,,\\ x^2 &= \sqrt{R^2+k}\sin\theta_1\sin\phi_1\,,\\ x^3 &= R\cos\theta_1\cos\phi_2\,,\\ x^4 &= R\cos\theta_1\sin\phi_2\,. \end{aligned} \tag{5.14}$$

[5]One could introduce a second constant in x^3 and x^4 but it can be removed by shifting the 'radial' coordinate.

In this case, the flat space metric becomes

$$
\begin{aligned}
ds^2 = & \frac{R^2 + k\cos^2\theta_1}{R^2+k} dR^2 + (R^2 + k\cos^2\theta_1) d\theta_1^2 + \\
& + (R^2+k)\sin^2\theta_1 d\phi_1^2 + R^2\cos^2\theta_1 d\phi_2^2
\end{aligned} \tag{5.15}
$$

and the (relevant) solution of Laplace's equation depending only on R is

$$
f(R) = \frac{a}{k}\log\frac{R^2+k}{R^2}\,. \tag{5.16}
$$

We can again normalize a by comparing to the spherical solution (2.5) at large radius. Hence for a system of N wrapped D5-branes, we find

$$
a = N g_s \ell_s^2\,. \tag{5.17}
$$

The brane density is:

$$
\rho_4 = \frac{N}{2\pi^2}\frac{1}{\sqrt{R^2(R^2+k)(R^2+k\cos^2\theta_1)}}\left(H_{p+4}^{\alpha_2}H_p^{\beta_2}\right)^{-3/2}\Big|_{R=R_e}\,. \tag{5.18}
$$

Note that for positive (negative) k, the brane density is most concentrated near the x^1-x^2 (x^3-x^4) plane.

There is a similar construction for five transverse dimensions, i.e., with $p = 0$ and wrapped D4-branes, but the solution is expressed in terms of elliptic integrals.

6. Discussion

In this paper, we have investigated the construction of supergravity solutions describing enhançons which are not spherically symmetric. A simple extension of the usual probe calculation [1] confirms that the enhançon locus occurs where the K3 volume reaches the string scale volume V_*, in general. Further analysis shows that interior to this surface, the repulson geometry should be excised and replaced with ordinary flat space. We confirmed that the boundary shell in the resulting solution acts as a shell of wrapped D$(p+4)$-branes smeared out across the enhançon locus. That is, we showed that the shell acts as a source for the metric, dilaton and RR fields precisely in the way a collection of wrapped D$(p+4)$-branes should. We also presented some explicit examples of nonspherical enhançon solutions.

In that these constructions involve solving Laplace's equation with certain boundary conditions, the analysis is reminiscent of ordinary electrostatics. In this analogy, the enhançon shell and the interior region have a fixed 'potential' and so behave like a lump of conducting material. Further, the brane density (4.5) is proportional to the magnitude of the gradient of the 'potential', i.e., the 'electric field', at the surface, as expected for the 'charge' density. In particular, our intuition from electrostatics would say that the branes arrange themselves on a curved enhançon shell so as to be concentrated in the regions where the curvature of the shell is greatest. This intuition was confirmed for the explicit examples discussed in section 5.

In the present case though, for a particular solution to be physically sensible, we must ensure that the 'charge'/brane density is everywhere positive on the enhançon shell. While this may seem to be true by definition in eq. (4.5), let us consider the solutions constructed in subsection 5.1. Note that we restricted our analysis there to consider the higher spherical harmonic as a perturbation of the spherically symmetric system (i.e., we assumed $a/r_e^{3-p+L} \ll 1$). However, the harmonic function given in eq. (5.1) provides a solution of the full supergravity equations for any amplitude of the higher harmonic. However, as a is increased, eventually we will find that the gradient of f vanishes at certain positions on the enhançon locus, i.e., the brane density vanishes at these points. For larger values of a, when we approach the origin from certain directions, f will reach a maximum which is less than f_* and then eventually becomes negative closer to the origin. While these configurations still solve the supergravity equations, these solutions are pathological and singular, and they should be discarded as being unphysical.

A different restriction arises for these solutions of subsection 5.1 if we wish to limit ourselves to supergravity solutions which reliably describe the physics. That is, we may ask that the effective wavelength of a perturbation on the sphere should be no smaller thatn the spacing between constituent branes in the enhançon. This requirement puts an upper bound on the angular quantum number of the higher harmonic. For example, with $p = 2$ (wrapped D6-branes), the brane density for the spherical enhançon (4.6) reduces to

$$\rho_{sph} = \frac{N}{4\pi r_e^2} H_{p+4}^{-\alpha_2} H_p^{-\beta_2} = \frac{1}{d^2}\,, \tag{6.1}$$

which defines a brane spacing of d. Now, the effective wavelength of the higher harmonic is simply the proper circumference of the equator of the enhançon sphere divided by the angular quantum number L, i.e.,

$$\lambda = \frac{2\pi r_e}{L} H_{p+4}^{\alpha_2/2} H_p^{\beta_2/2}\,. \tag{6.2}$$

Hence with $d \lesssim \lambda$, we find the upper bound that $L^2 \lesssim \pi N$. With similar analysis for $p = 1, 0$, we can generalize this result to $L^{4-p} \lesssim N$.

Now, the number of perturbations which we can reliably study here essentially counts the dimension of the moduli space of the N-charge enhançon which is accessible using the present supergravity techniques. Focussing again on $p = 2$ and ordinary spherical harmonics in S^2, we know that the dimension of eigenspace of the laplacian with eigenvalue $L(L+1)$ is $2L+1$. Hence summing these dimensions gives

$$\sum_{L=1}^{L_{\max}} = L_{\max}(L_{\max}+2)\,. \tag{6.3}$$

Hence the dimension of the relevant portion of the moduli space is roughly $\pi\, N$. Similarly for general p, one finds that the relevant portion of the moduli space has roughly dimension N in all cases. Perhaps these results should not be seen as very surprising since essentially we are saying that within this framework we have control over the position moduli of the individual branes, which would give $(5-p)N$ parameters in the general case.

Comparing the $p = 2$ case to that of BPS magnetic monopoles in the SU(2) gauge theory, we know that the dimension of the full moduli space for the latter is $4N - 1$ [11]. Essentially this is comprised of the $3N$ 'position' parameters for the individual monopoles and $N-1$ relative gauge rotations. It is clear that the latter parameters will not be captured in the supergravity calculations. However, describing the role of the 'position' parameters becomes more complicated when the monopoles are very close together, and we see here that supergravity can capture some of these complications as the wrapped D6-branes merge together in a macroscopic enhançon configuration. To get a better description of the microscopic details would require explicitly including the gauge field degrees of freedom in the low energy effective theory, perhaps along the lines of ref. [8].

The origin of the enhançon is in the enhanced gauge symmetry appearing, or a particular vector supermultiplet becoming massless when the internal geometry enters a string regime. Denef [23] found a similar effect in N=1 theories where a hypermultiplet becomes massless at a particular point in the moduli space of the internal Calabi-Yau space, and he denoted the analogous solutions as 'empty holes'. Of course, it would be possible to extend the present calculations to that particular framework. Another simple extension would be to include fundamental Dp-branes or other SUSY preserving branes (or momentum modes) into the configuration, along the lines of [7, 14].

Acknowledgments

This research was supported by NSERC of Canada and Fonds FCAR du Québec. RCM would like to thank the Institute for Theoretical Physics at UCSB for hospitality during part of this work. DA would like to thank the Perimeter Institute for Theoretical Physics and the University of Waterloo's Department of Physics for their kind hospitality during this work. Research at the ITP was supported in part by the U.S. National Science Foundation under Grant No. PHY99–07949. We would like to thank Clifford Johnson, Amanda Peet and David Winters for useful conversations and correspondence. In particular, we thank Clifford Johnson for informing us of ref. [24], which discusses some interesting examples of nonspherical enhançons. Finally, we thank Frédéric Leblond and David Winters for proof-reading an earlier draft of this paper.

A. Harmonic functions in $\mathbb{R}^N$

We would like to find the general solution of Laplace's equation on R^N in spherical polar coordinates.[6] In the problem at hand, N is the dimension of the transverse space, i.e., $N = 5 - p$ for the wrapped D$(p + 4)$-brane. We choose polar coordinates so that the flat space metric becomes

$$ds^2 = dr^2 + r^2 \left(d\theta_{N-2}^2 + \sin^2\theta_{N-2}\left(d\theta_{N-3}^2 + \cdots + \sin^2\theta_2\left(d\theta_1^2 + \sin^2\theta_1\, d\phi^2\right)\right)\right). \qquad \text{(A.1)}$$

[6]We are grateful to David Winters for his advice on these solutions.

With this choice, Laplace's equation becomes

$$
\begin{aligned}
0 = \nabla^2 f &= \frac{1}{\sqrt{\hat{g}}}\partial_i\left(\sqrt{\hat{g}}\hat{g}^{ij}\partial_j f\right) \\
&= \frac{1}{r^{N-1}}\left(r^{N-1}\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\left[\sum_{n=1}^{N-2}\frac{1}{\sin^2\theta_{N-2}\cdots\sin^2\theta_{n+1}\sin^n\theta_n}\frac{\partial}{\partial\theta_n}\left(\sin^n\theta_n\frac{\partial f}{\partial\theta_n}\right) + \right. \\
&\qquad\qquad \left. + \frac{1}{\sin^2\theta_{N-2}\cdots\sin^2\theta_1}\frac{\partial^2 f}{\partial\phi^2}\right]. \qquad \text{(A.2)}
\end{aligned}
$$

Now we apply separation of variables with the ansatz

$$f(x^i) = R(r)\psi_{N-2}(\theta_{N-2})\ldots\psi_1(\theta_1)\psi_0(\phi) \qquad \text{(A.3)}$$

in which case eq. (A.2) can be separated into the following system of N ordinary differential equations

$$
\begin{aligned}
\frac{d^2\,\psi_0}{d\phi^2} + m^2\psi_0 &= 0 && \text{(A.4)} \\
\frac{d^2\psi_1}{d\theta_1^2} + \frac{\cos\theta_1}{\sin\theta_1}\frac{d\psi_1}{d\theta_1} + \left[l_1(l_1+1) - \frac{m^2}{\sin^2\theta_1}\right]\psi_1 &= 0 && \text{(A.5)} \\
\frac{d^2\psi_n}{d\theta_n^2} + n\frac{\cos\theta_n}{\sin\theta_n}\frac{d\psi_n}{d\theta_n} + \left[l_n(l_n+n) - \frac{l_{n-1}(l_{n-1}+1)}{\sin^2\theta_n}\right]\psi_n &= 0 && \text{(A.6)} \\
\frac{d^2R}{dr^2} + \frac{N-1}{r}\frac{dR}{dr} - \frac{l_{N-2}(l_{N-2}+N-2)}{r^2}R &= 0 && \text{(A.7)}
\end{aligned}
$$

where the index runs over $n = 2, \ldots, N-2$ in eq. (A.6). The angular equations for θ_n are easily solved if we make the change of variables $x_i = \cos\theta_i$, in which case these equations take the general form

$$(1-x^2)\frac{d^2\psi_{abc}}{dx^2} - a\,x\frac{d\psi_{abc}}{dx} + \left[b(b+a-1) - \frac{c(c+a-2)}{1-x^2}\right]\psi_{abc} = 0\,, \qquad \text{(A.8)}$$

where a, b and c are all integers. The relevant (normalizable) solutions may be written as

$$\psi_{abc}(x) = \frac{\Gamma(b+\frac{1}{2})}{\sqrt{\pi}\Gamma(b+\frac{a}{2}-\frac{1}{2})}(1-x^2)^{\frac{c}{2}}\frac{d^{b+c}}{dx^{b+c}}\sum_{n=0}^{b}\frac{(-1)^n\,2^{b-2n}}{n!(2b-2n)!}\Gamma\left(b+\frac{a}{2}-n-\frac{1}{2}\right)x^{2b-2n}\,. \qquad \text{(A.9)}$$

Eq. (A.4) has real solutions

$$\psi_0(\phi) = A\,e^{im\phi} + A^*\,e^{-im\phi}\,, \qquad \text{(A.10)}$$

while the general radial function solving eq. (A.7) takes the form

$$R(r) = \frac{B}{r^{N-2+l_{N-2}}} + C\,r^{l_{N-2}}\,. \qquad \text{(A.11)}$$

For the present problem, we are generally interested in localized solutions, i.e., $f \to 0$ as $r \to \infty$, and so we would set $C = 0$. Now the general solution of the Laplace equation in N dimensions is

$$f(r,\theta_n,\phi) = \sum_{l_{N-2}=0}^{\infty} \cdots \sum_{l_1=0}^{l_2} \sum_{m=0}^{l_1} R_{l_{N-2}}(r)(Ae^{im\phi} + A^* e^{-im\phi})\psi_{2\,l_1\,m}(\cos\theta_1) \times \\ \times \psi_{3\,l_2\,l_1}(\cos\theta_2) \cdots \psi_{N-1\,l_{N-2} l_{N-3}}(\cos\theta_{N-2}) \ . \quad \text{(A.12)}$$

References

[1] C.V. Johnson, A.W. Peet and J. Polchinski, *Gauge theory and the excision of repulson singularities*, *Phys. Rev.* **D 61** (2000) 086001 [hep-th/9911161].

[2] R.C. Myers, *Dielectric-branes*, *J. High Energy Phys.* **12** (1999) 022 [hep-th/9910053].

[3] K. Behrndt, *About a class of exact string backgrounds*, *Nucl. Phys.* **B 455** (1995) 188 [hep-th/9506106].

[4] R. Kallosh and A.D. Linde, *Exact supersymmetric massive and massless white holes*, *Phys. Rev.* **D 52** (1995) 7137 [hep-th/9507022].

[5] See also, M. Cvetič and D. Youm, M. Cvetič and D. Youm, *Singular BPS saturated states and enhanced symmetries of four-dimensional $N = 4$ supersymmetric string vacua*, *Phys. Lett.* **B 359** (1995) 87 [hep-th/9507160].

[6] C.V. Johnson, *Enhancons, fuzzy spheres and multi-monopoles*, *Phys. Rev.* **D 63** (2001) 065004 [hep-th/0004068]; *The enhancon, multimonopoles and fuzzy geometry*, *Int. J. Mod. Phys.* **A 16** (2001) 990 [hep-th/0011008].

[7] C.V. Johnson, R.C. Myers, A.W. Peet and S.F. Ross, *The enhancon and the consistency of excision*, *Phys. Rev.* **D 64** (2001) 106001 [hep-th/0105077].

[8] M. Wijnholt and S. Zhukov, *Inside an enhancon: monopoles and dual Yang-Mills theory*, hep-th/0110109.

[9] M. Krogh, *S-duality and tensionless 5-branes in compactified heterotic string theory*, *J. High Energy Phys.* **12** (1999) 018 [hep-th/9911084];
M. Natsuume, *The heterotic enhancon*, hep-th/0111044.

[10] E.J. Weinberg and A.H. Guth, *Nonexistence of spherically symmetric monopoles with multiple magnetic charge*, *Phys. Rev.* **D 14** (1976) 1660.

[11] E. Corrigan and P. Goddard, *An n monopole solution with $4n - 1$ degrees of freedom*, *Commun. Math. Phys.* **80** (1981) 575;
N.J. Hitchin, *Monopoles and geodesics*, *Commun. Math. Phys.* **83** (1982) 579; *On the construction of monopoles*, *Commun. Math. Phys.* **89** (1983) 145.

[12] W. Israel, *Singular hypersurfaces and thin shells in general relativity*, *Nuovo Cim.* **B44S10** (1966) 1, erratum *ibid.* **B48** (1967) 463.

[13] See also C.W. Misner, K.S. Thorne and J.A. Wheeler, *Gravitation* San Francisco, Freeman 1973.

[14] C.V. Johnson and R.C. Myers, *The enhancon, black holes and the second law*, *Phys. Rev.* **D 64** (2001) 106002 [hep-th/0105159].

JHEP02(2002)043

[15] N.R. Constable, *The entropy of 4d black holes and the enhancon*, *Phys. Rev.* **D 64** (2001) 104004 [hep-th/0106038].

[16] C.V. Johnson, *D-brane primer*, hep-th/0007170.

[17] A.W. Peet, *Tasi lectures on black holes in string theory*, hep-th/0008241.

[18] M. Bershadsky, C. Vafa and V. Sadov, *D-strings on D-manifolds*, *Nucl. Phys.* **B 463** (1996) 398 [hep-th/9510225].

[19] M.B. Green, J.A. Harvey and G.W. Moore, *I-brane inflow and anomalous couplings on D-branes*, *Class. and Quant. Grav.* **14** (1997) 47 [hep-th/9605033];
Y.-K.E. Cheung and Z. Yin, *Anomalies, branes and currents*, *Nucl. Phys.* **B 517** (1998) 69 [hep-th/9710206].

[20] K. Dasgupta, D.P. Jatkar and S. Mukhi, *Gravitational couplings and Z_2 orientifolds*, *Nucl. Phys.* **B 523** (1998) 465 [hep-th/9707224].

[21] C.P. Bachas, P. Bain and M.B. Green, *Curvature terms in D-brane actions and their M-theory origin*, *J. High Energy Phys.* **05** (1999) 011 [hep-th/9903210].

[22] See, for example G. Arfken, *Mathematical methods for physicists*, 2nd ed., New York, Academic Press 1970.

[23] F. Denef, *Supergravity flows and D-brane stability*, *J. High Energy Phys.* **08** (2000) 050 [hep-th/0005049].

[24] L.M. Dyson, L. Järv and C.V. Johnson, *Oblate, toroidal and other shapes for the enhancon*, hep-th/0112132.